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Introduction to Time Series and Forecasting

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Appendix D, and a new Section 11.5 provides an introduction to continuous parameter ARMA (or CARMA) processes.

The diskette containing the student version of the package ITSM2000 is no longer included with the book since the professional version (which places no limit on the length of the series to be studied) can now be downloaded from <http://extras.springer.com> as indicated above. A tutorial for the use of the package is provided as Appendix E and a searchable file, ITSM_HELP.pdf, giving more detailed instructions, is included with the package.

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- 1.2 Objectives of Time Series Analysis
- 1.3 Some Simple Time Series Models
- 1.4 Stationary Models and the Autocorrelation Function
- 1.5 Estimation and Elimination of Trend and Seasonal Components
- 1.6 Testing the Estimated Noise Sequence

In this chapter we introduce some basic ideas of time series analysis and stochastic processes. Of particular importance are the concepts of stationarity and the autocovariance and sample autocovariance functions. Some standard techniques are described for the estimation and removal of trend and seasonality (of known period) from an observed time series. These are illustrated with reference to the data sets in Section 1.1. The calculations in all the examples can be carried out using the time series package ITSM, the professional version of which is available at <http://extras.springer.com>. The data sets are contained in files with names ending in .TSM. For example, the Australian red wine sales are filed as WINE.TSM. Most of the topics covered in this chapter will be developed more fully in later sections of the book. The reader who is not already familiar with random variables and random vectors should first read Appendix A, where a concise account of the required background is given.

1.1 Examples of Time Series

A **time series** is a set of observations x_t , each one being recorded at a specific time t . A *discrete-time time series* (the type to which this book is primarily devoted) is one in which the set T_0 of times at which observations are made is a discrete set, as is the case, for example, when observations are made at fixed time intervals. *Continuous-time time series* are obtained when observations are recorded continuously over some time interval, e.g., when $T_0 = [0, 1]$.